



ARIZONA EDUCATOR PROFICIENCY ASSESSMENTS®



STUDY GUIDE

37 Middle Grades Mathematics

This AEPA test was replaced by a NES test. Examinees may continue to find this study guide useful as they prepare for the NES, as the previous AEPA test may have covered objectives and content similar to the NES test.

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PART 1: GENERAL INFORMATION ABOUT THE AEPA® AND TEST PREPARATION

Part 1 of this study guide is contained in a separate PDF file. Click the link below to view or print this section:

[General Information About the AEPA and Test Preparation](#)



PART 2: FIELD-SPECIFIC INFORMATION

Field 37: Middle Grades Mathematics

INTRODUCTION

This section includes a list of the test objectives, practice questions, and an answer key for the selected-response questions.

Test objectives. As noted earlier, the test objectives are broad, conceptual statements that reflect the knowledge, skills, and understanding an entry-level educator needs to practice effectively in Arizona schools. The list of test objectives for each test field is the *only* source of information about what a specific test will cover and therefore should be studied carefully.

Practice questions. The practice questions for the selected-response and performance assignment sections included in this section are designed to give you an introduction to the nature of the questions included in the AEPA® tests. The practice questions represent the various types of questions you may expect to see on an actual test; however, they are *not* designed to provide diagnostic information to help you identify specific areas of individual strength or weakness or to predict your performance on the test as a whole.

When you answer the practice questions, you may wish to use the sample answer sheet and sample Written Response Booklet provided in Part 1 to acquaint yourself with these materials. Use the answer key located after the practice questions to check your answers. A sample response is provided immediately following the written performance assignment. The sample response in this guide is for illustrative purposes only. Your written response should be your original work, written in your own words, and not copied or paraphrased from some other work.

To help you identify how the test objectives are measured, the objective statement to which the question corresponds is listed in the answer key. When you are finished with the practice questions, you may wish to go back and review the entire list of test objectives and descriptive statements for your test field.

Preparation resources. The list of preparation resources has been compiled to assist you in finding relevant materials as you prepare to take the Middle Grades Mathematics test. This list is to be considered not as complete, but as representative of the kinds of resources currently available. There may be other materials that may be helpful to you in preparing to take the test.

You may also wish to consult a representative from an Arizona educator preparation program in your area regarding other potential resources specific to this field. Keep in mind that the use of these materials does not guarantee successful performance on the test.

TEST OBJECTIVES

Field 37: Middle Grades Mathematics

SUBAREAS:

1. Number Sense and Operations
2. Data Analysis, Probability, and Discrete Mathematics
3. Patterns, Algebra, and Functions
4. Geometry and Measurement
5. Mathematical Processes and Reasoning

NUMBER SENSE AND OPERATIONS

0001 Understand the structure of numeration systems and ways of representing numbers.

For example: classifying real numbers as natural, whole, integer, rational, or irrational; determining equivalency between and among fractions, decimals, and percents in contextual situations; using different representations of numbers (e.g., powers, roots, scientific notation); and solving problems involving integers, fractions, decimals, percents, ratios, and proportions.

0002 Understand numerical operations and properties of the real number system.

For example: simplifying expressions using order of operations; identifying and applying properties of the real number system (e.g., distributive, associative); analyzing numerical operations and their inverses; solving word problems using numerical operations; and using appropriate techniques of estimation to solve number problems.

0003 Understand the principles of number theory.

For example: demonstrating an understanding of concepts from number theory (e.g., greatest common factor, least common multiple); analyzing properties of prime and composite numbers; determining prime factorization of numbers; and solving problems using basic concepts from number theory.

DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS

0004 Understand methods of collecting, organizing, and displaying data.

For example: demonstrating knowledge of ways to formulate questions to collect data in contextual situations; analyzing and using appropriate sampling techniques; using and interpreting charts and graphs (e.g., circle graphs, line graphs, stem-and-leaf plots, box plots, scatter plots) to organize and display data; and identifying a linear function that best represents a set of data.

0005 Understand methods of describing and analyzing data.

For example: determining and interpreting measures of central tendency (i.e., mean, median, mode) and dispersion (e.g., range, standard deviation); recognizing the effects of data transformations on measures of central tendency and dispersion; making appropriate inferences, interpolations, and extrapolations from a set of data; and interpreting percentiles and correlation.

0006 Understand the fundamental principles of probability, probability distributions, and discrete mathematics.

For example: computing probabilities involving dependent, independent, and mutually exclusive events; computing conditional probabilities; applying properties of uniform and normal distributions; creating and using probability models to describe real-world situations; and solving counting problems using tables, tree diagrams, permutations, and combinations.

PATTERNS, ALGEBRA, AND FUNCTIONS**0007 Understand patterns, relations, algebraic functions, and expressions.**

For example: analyzing and extending iterative and recursive patterns using symbols or numbers; manipulating and simplifying algebraic expressions; analyzing characteristics of algebraic functions (e.g., domain, range, dependent versus independent variables); using qualitative graphing; and analyzing relationships among tabular, graphic, and algebraic representations of functions.

0008 Understand and use linear functions to model and solve problems.

For example: demonstrating knowledge of linear functions using various representations (e.g., tabular, graphical, symbolic, verbal); determining the equation of a line in a variety of situations; analyzing the relationship between linear models and rates of change; and modeling and solving problems involving linear equations, inequalities, and systems using a variety of methods (e.g., algebraic, graphical).

0009 Understand and use nonlinear functions and relations to model and solve problems.

For example: distinguishing between relations and functions; solving quadratic equations using a variety of methods (e.g., factoring, quadratic formula); solving quadratic inequalities using different methods (e.g., graphing); analyzing properties and graphs of polynomial, radical, rational, absolute value, and exponential functions; and modeling and solving problems involving nonlinear functions.

GEOMETRY AND MEASUREMENT**0010 Understand principles and procedures related to measurement.**

For example: selecting and using appropriate units of measurement for a contextual situation; using and converting within and between different measurement systems; solving problems involving length, perimeter, area, volume, mass, angles, time, temperature, and speed; and determining the effects of changing linear dimensions on measures of perimeter, area, and volume.

0011 Understand principles of Euclidean geometry and properties of two- and three-dimensional figures.

For example: applying concepts and properties of points, lines, angles, and planes; applying geometric concepts (e.g., similarity, congruence, parallelism); analyzing and solving problems involving two- and three-dimensional figures (e.g., polygons, circles, prisms, cylinders, cones); and demonstrating knowledge of cross sections and nets of three-dimensional figures.

0012 Understand principles of coordinate and transformational geometry.

For example: representing geometric figures in the coordinate plane; applying geometric concepts (e.g., distance, midpoint, slope, parallel and perpendicular lines) to classify and analyze figures; applying transformations (e.g., translations, rotations, reflections, dilations); and using coordinate and transformational geometry to model and solve problems.

MATHEMATICAL PROCESSES AND REASONING

0013 Understand principles of problem solving.

For example: distinguishing between necessary and unnecessary information in word problems; demonstrating knowledge of developing formulas to calculate areas and volumes of simple geometric figures; applying appropriate reasoning to solve mathematical problems in contextual situations; and analyzing algorithms and formulas to determine their purpose or validity.

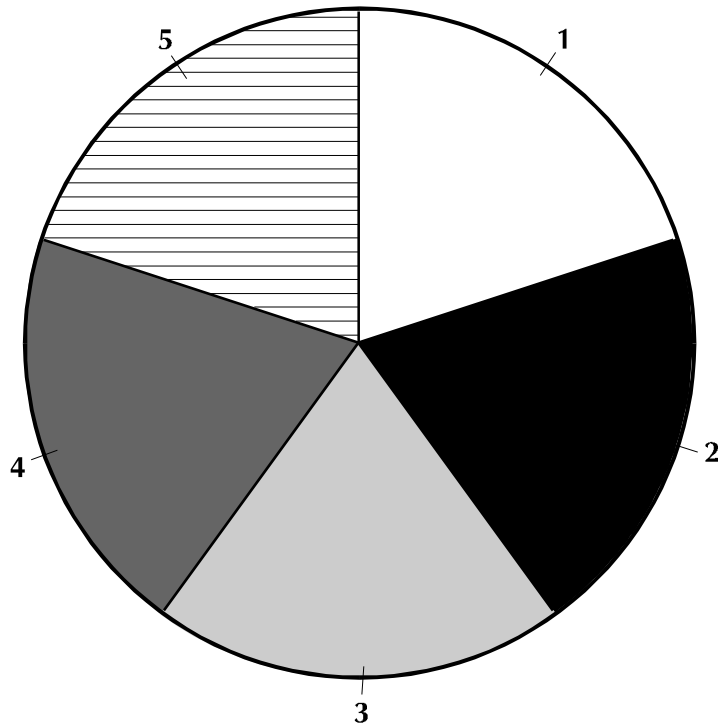
0014 Understand principles of mathematical reasoning.






For example: using counterexamples and principles of logic to evaluate arguments; understanding inductive and deductive reasoning; understanding the components of a formal argument (e.g., if...then statements); and demonstrating knowledge of evaluating the validity of an argument.

0015 Understand the techniques for communicating mathematical information and connections within and outside the domains of mathematics.

For example: using a variety of mathematical representations (e.g., graphic, symbolic, verbal) to communicate mathematical ideas and concepts; recognizing and analyzing connections among different mathematical concepts (e.g., geometry and algebra); and applying mathematics to solve problems in other disciplines and in everyday situations.

DISTRIBUTION OF SELECTED-RESPONSE ITEMS ON THE TEST FORM



Subarea	Approximate Percentage of Selected-Response Items on Test Form
 1. Number Sense and Operations	20%
 2. Data Analysis, Probability, and Discrete Mathematics	20%
 3. Patterns, Algebra, and Functions	20%
 4. Geometry and Measurement	20%
 5. Mathematical Processes and Reasoning	20%

FORMULAS

Formula	Description
$V = \frac{1}{3}Bh$	Volume of a right cone and a pyramid
$A = 4\pi r^2$	Surface area of a sphere
$V = \frac{4}{3}\pi r^3$	Volume of a sphere
$A = \pi r\sqrt{r^2 + h^2}$	Lateral surface area of a right circular cone
$S_n = \frac{n}{2}[2a + (n - 1)d] = n\left(\frac{a + a_n}{2}\right)$	Sum of an arithmetic series
$S_n = \frac{a(1 - r^n)}{1 - r}$	Sum of a geometric series
$\sum_{n=0}^{\infty} ar^n = \frac{a}{1 - r}, r < 1$	Sum of an infinite geometric series
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance formula
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	Midpoint formula
$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$	Slope
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	Law of sines
$c^2 = a^2 + b^2 - 2ab \cos C$	Law of cosines
$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}$	Variance
$s = r\theta$	Arc length
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic formula

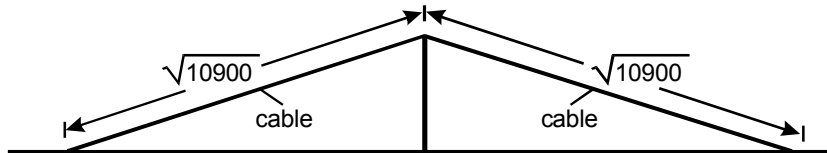
PRACTICE QUESTIONS

Field 37: Middle Grades Mathematics

Examinees taking the Middle Grades Mathematics test will be provided with one of the following models of scientific calculators at the test administration. The models distributed are subject to change; directions for use will not be provided at the test site. You may not use your own calculator or calculator manual for this test.

- Texas Instruments TI-30X
- Texas Instruments TI-30X Solar
- Texas Instruments TI-30Xa
- Texas Instruments TI-30Xs
- Texas Instruments TI-30XIIs

1. Use the diagram below to answer the question that follows.



The suspension bridge in the diagram above uses two steel cables, each with a length of $\sqrt{10900}$ feet. The total weight of the two cables is 5000 pounds. Which of the following expressions represents the number of pounds per foot of cable?

- A. $\frac{2.5\sqrt{109}}{109}$
- B. $\frac{5\sqrt{109}}{109}$
- C. $\frac{250\sqrt{109}}{109}$
- D. $\frac{500\sqrt{109}}{109}$

2. In a retail store, the owner earns $\frac{1}{4}$ more than the manager earns. The manager is earning what percentage of the owner's salary?
- A. 75%
 - B. 80%
 - C. 120%
 - D. 125%

3. **Use the problem below to answer the question that follows.**

A caterer prepares lunch plates for people attending a conference. Each plate comes with three cookies for dessert. What is the maximum number of lunch plates that can be made with 100 cookies?

Which of the following descriptions most narrowly defines the maximum number of plates in the problem above?

- A. a rational number
- B. a natural number
- C. an irrational number
- D. an integer

4. There is a 4 to 3 ratio of girls to boys in the neighborhood. If g represents the number of girls and b represents the number of boys, which of the following equations represents the relationship between g and b ?
- A. $b = \frac{3}{7}g$
- B. $b = \frac{4}{7}g$
- C. $b = \frac{3}{4}g$
- D. $b = \frac{4}{3}g$
5. The fourth-quarter gross domestic product for a particular year was \$12,497,400,000,000. The population for the same year was 2.9895×10^8 . If the gross domestic product is divided by the population, the result is called the per capita gross domestic product. Which of the following correctly uses scientific notation to express the approximate per capita gross domestic product for the data above?
- A. 0.41804×10^4 dollars
- B. 4.1804×10^3 dollars
- C. 0.41804×10^5 dollars
- D. 4.1804×10^4 dollars
6. Which of the following expressions is equivalent to $\sqrt{900}$?
- A. $100\sqrt{9}$
- B. $(\sqrt{300})^2$
- C. $\sqrt{4} \cdot \sqrt{9} \cdot \sqrt{25}$
- D. $\sqrt{300} + \sqrt{300} + \sqrt{300}$

7. Use the bill of sale below to answer the question that follows.

<u>Bill of Sale</u>	
salad 2 @	\$ 4.75
steak 2 @	\$15.25
cake 2 @	\$ 3.95

Two friends went out to dinner and ordered the items listed on the bill of sale shown above. Which of the following expressions best represents a procedure for using mental math to estimate a 15% tip for the meal?

- A. $\$47.90(0.15)$
- B. $\$48(15)$
- C. $\$4.79 + \frac{1}{2}(\$4.79)$
- D. $\$4.80 + \frac{1}{2}(\$4.80)$
8. Which of the following equations demonstrates an associative property of operations?

A. $2\left(\frac{3+5}{4}\right) = \frac{2 \cdot 3 + 2 \cdot 5}{4}$

B. $\frac{3}{4} + \left(\frac{1}{4} + \frac{1}{9}\right) = \left(\frac{3}{4} + \frac{1}{4}\right) + \frac{1}{9}$

C. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{6}{12} + \frac{4}{12} + \frac{3}{12}$

D. $\frac{2}{3} + \frac{3}{4} + \frac{1}{3} + \frac{1}{4} = \frac{2}{3} + \frac{1}{3} + \frac{3}{4} + \frac{1}{4}$

9. A child received some money as a birthday gift. The child placed $\frac{1}{3}$ of the money in a savings account. Of the remainder, $\frac{1}{3}$ was used to buy a movie ticket, $\frac{1}{4}$ was used to buy popcorn, and \$4 were used to buy a drink. The child was left with 6 remaining dollars. What was the original total value of the birthday gift?
- A. \$36
- B. \$24
- C. \$18
- D. \$15
10. **Use the expression below to answer the question that follows.**

$$\frac{3}{7} + \frac{1}{3}$$

Which of the following fractions, when added to the expression above, results in a sum equal to 0?

- A. $\frac{(-3)(-7)}{-(3^2 + 7)}$
- B. $\frac{(3)(7)}{3^2 + 7}$
- C. $\frac{-3^2 - 7}{(3)(7)}$
- D. $\frac{1 - 3^2 - 7}{(3)(7)}$

11. Use the problem and solution below to answer the question that follows.

Problem:

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - (4)(-3)(6)}}{(2)(-3)}$$

Solution:

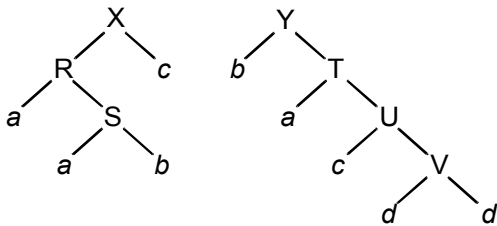
step 1	$x = \frac{2 \pm \sqrt{(4 - 4)(-3)(6)}}{-6}$
step 2	$x = \frac{2 \pm \sqrt{(0)(-3)(6)}}{-6}$
step 3	$x = \frac{2 \pm \sqrt{0}}{-6}$
step 4	$x = -\frac{1}{3}$

A student performed the steps above to solve the problem. In which step did the student introduce an error?

- A. step 1
- B. step 2
- C. step 3
- D. step 4

12. Aquaculture researchers are using an automated feeding system to conduct an experiment on the optimal feeding cycle for a species of fish. One group of fish is fed every 12 hours, one group is fed every 8 hours, and one group is fed every 5 hours. If all three groups of fish are fed for the first time at 12:00 noon on January 1, when will they next all be fed at the same time?
- A. 12:00 noon on January 5
 - B. 12:00 noon on January 6
 - C. 12:00 noon on January 20
 - D. 12:00 noon on January 21

13. Use the prime factorization trees below to answer the question that follows.



Which of the following expressions gives the least common multiple of X and Y?

- A. $a \cdot b \cdot c \cdot d$
- B. $a \cdot b \cdot c \cdot d^2$
- C. $a^2 \cdot b \cdot c \cdot d^2$
- D. $a^3 \cdot b^2 \cdot c^2 \cdot d^2$

14. The prime factorization of $9!$ is represented by which of the following expressions?

A. $2 \cdot 3 \cdot 5 \cdot 7$
B. $2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9$
C. $2^3 \cdot 3^2 \cdot 5 \cdot 7$
D. $2^7 \cdot 3^4 \cdot 5 \cdot 7$

15. Use the equations below to answer the question that follows.

$$m = 2^3 \cdot 3 \cdot 5^2$$

$$n = 2 \cdot 3^5 \cdot 7$$

Which of the following expressions represents the greatest common factor of m and n ?

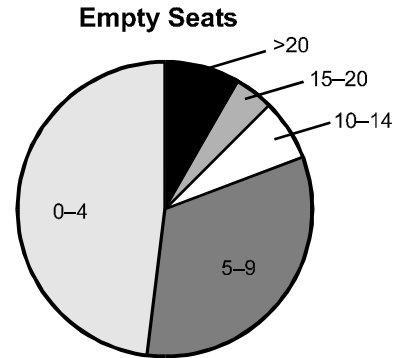
- A. $2 \cdot 3$
B. $2 \cdot 3 \cdot 5 \cdot 7$
C. $2^3 \cdot 3^5 \cdot 5^2 \cdot 7$
D. $2^4 \cdot 3^6 \cdot 5^2 \cdot 7$
16. Which of the following concepts is implicit in the process of adding fractions with different denominators?
- A. the greatest common factor
B. the least common multiple
C. the arithmetic mean
D. the cross product

17. An airline that flies regularly into Phoenix states that 48% of its flights have fewer than 5 empty seats. Which of the following data representations best supports this statement?

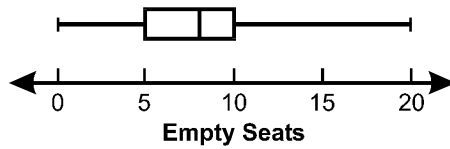
A.

Number of Empty Seats	Number of Flights
0–4	230
5–9	190
10–14	40
15–20	30
>20	10

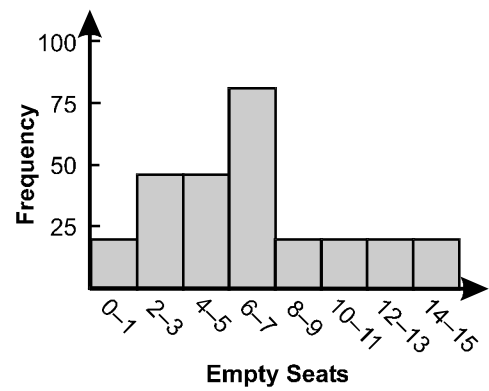
B.



C.



D.



18. Which of the following types of representations most clearly shows the median of a set of data?

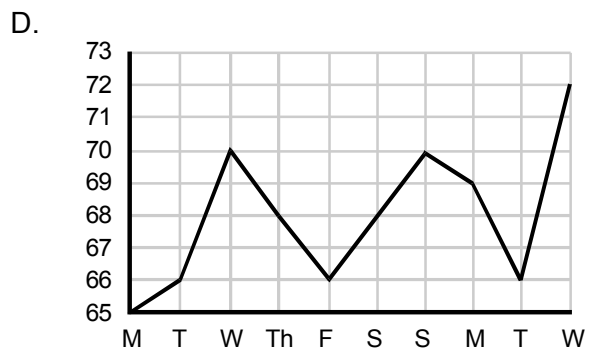
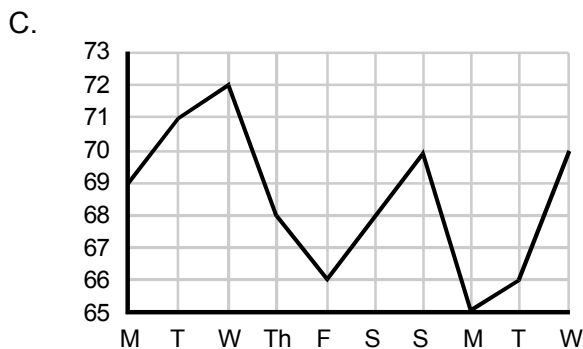
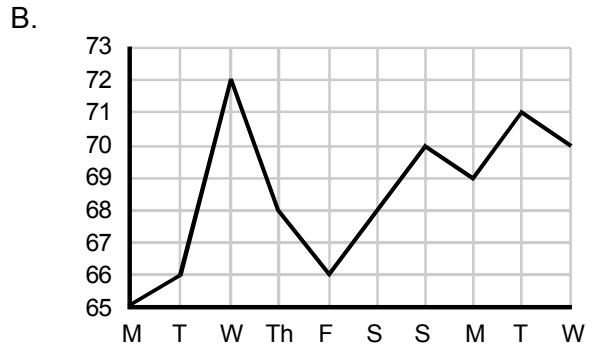
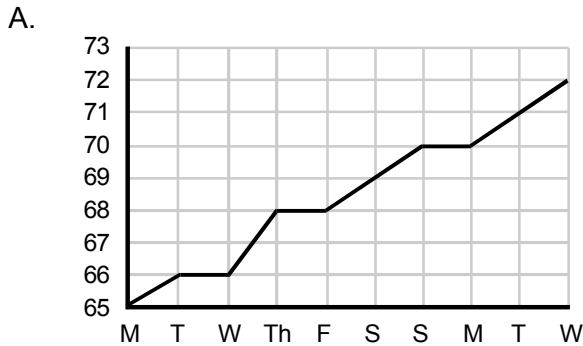
- A. scatter plot
- B. box plot
- C. histogram
- D. circle graph

19. Medical researchers are interviewing users of two drugs about the relative severity of the side effects associated with each of the medications. Which of the following questions is most likely to elicit information that will lead to valid conclusions?
- A. Do you agree very strongly, strongly, very little, or not at all with the statement, "Brand X causes less severe side effects than Brand Y"?
 - B. Considering the severity of their side effects, how would you rank the effectiveness of Brand X compared to Brand Y?
 - C. Researchers have concluded that Brand X causes less severe side effects than Brand Y. Do you agree with these conclusions?
 - D. In your experience, which medication had less severe side effects, Brand X or Brand Y?
20. A local government is collecting information on the number of children per family in a town. Which of the following sampling methods would produce the most reliable information?
- A. Survey 200 people coming out of a local grocery store on a Saturday morning.
 - B. Call 200 telephone numbers selected at random from the local telephone directory.
 - C. Analyze census data for 200 randomly selected local addresses.
 - D. E-mail a survey to the parents of 200 randomly selected children in the local public school system.

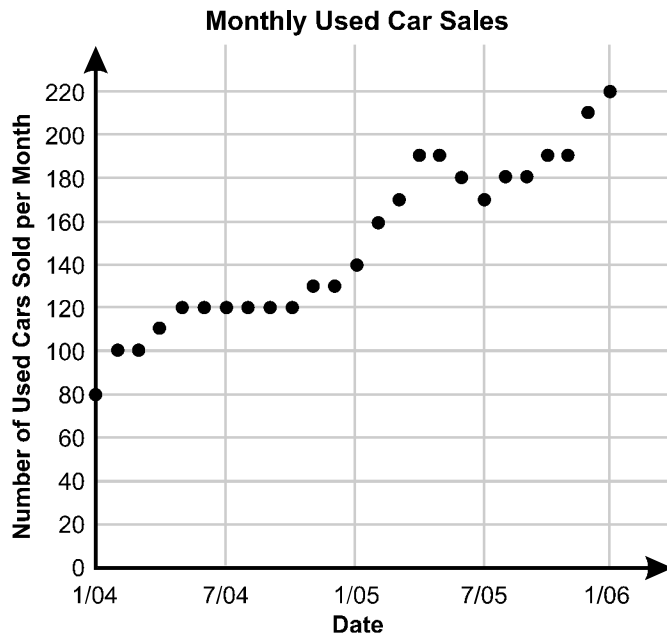
21. Use the table below to answer the question that follows.

High Temperature (°F)	Day of the Week
65	Monday the 1st
66	Tuesday the 2nd, Friday the 5th
67	
68	Thursday the 4th, Saturday the 6th
69	Monday the 8th
70	Sunday the 7th, Wednesday the 10th
71	Tuesday the 9th
72	Wednesday the 3rd

The table above shows the high temperatures for the first 10 days of the month. Which of the following graphs best represents the data in the table?



Use the graph below to answer the two questions that follow.



The graph above represents the number of used cars sold per month by a dealer from January 2004 to January 2006.

22. If y represents the number of used cars sold per month, and x represents months since January 2004, which of the following equations best fits the data?
- A. $y = 0.2x + 110$
 - B. $y = 0.2x + 90$
 - C. $y = 5x + 110$
 - D. $y = 5x + 90$
23. What is the mode of the number of cars sold per month for the time period shown?
- A. 120
 - B. 140
 - C. 150
 - D. 220

24. Use the chart below to answer the question that follows.

Class Scores on Final Exam

0	0	30	45	48
50	55	59	60	65
77	80	88	88	88
91	93	98	98	100

The chart above shows the class scores on a final exam. A student scored a 60 on the exam. Which is the best estimate of the student's percentile ranking in the class?

- A. 20th percentile
- B. 40th percentile
- C. 60th percentile
- D. 90th percentile

25. Use the table below to answer the question that follows.

Cups of Coffee Consumed per Day					
Worker #	Mon	Tue	Wed	Thu	Fri
1	1	1	1	2	2
2	1	1	1	2	3
3	1	1	1	3	3

The table above shows the number of cups of coffee consumed per day by three different workers over the course of a week. Which of the following statistical measures is the same for all three workers?

- A. mean
- B. median
- C. standard deviation
- D. range

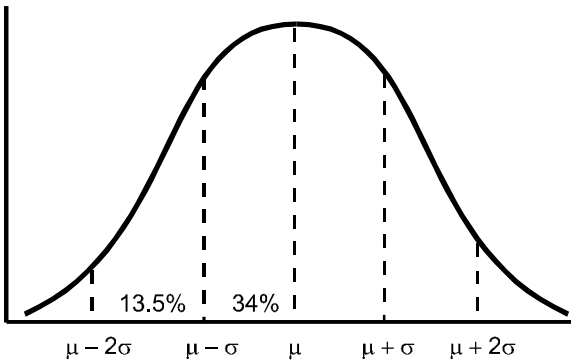
26. Use the table below to answer the question that follows.

	Year Number				
	1	2	3	4	5
Duck Pairs	32	36	34	31	35

The table above shows the number of mating pairs of ducks in a national park over a five-year period. The standard deviation of the data is 2.07. If there were twice as many mating pairs of ducks each year, what would the standard deviation be?

- A. $2.07 \div 2$
- B. 2.07
- C. $(2.07)^2$
- D. $2 \cdot 2.07$

Use the standard normal distribution and the information below to answer the two questions that follow.

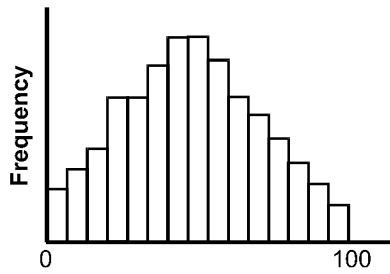


Medical researchers measured the body temperatures of 130 participants in a study. The participants consisted of a random selection of men, women, and children between the ages of 12 and 60. The researchers found that the body temperatures were approximately normally distributed with a mean of 98.25°F and a standard deviation of 0.73°F .

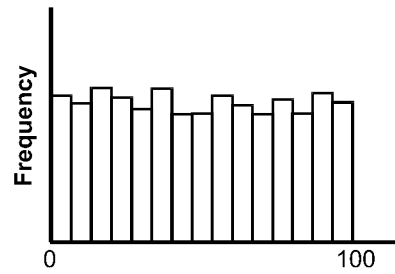
27. Approximately how many of the participants had body temperatures between 96.79°F and 98.98°F ?
- A. 88
 - B. 106
 - C. 109
 - D. 124
28. If a similar study were conducted using a more homogeneous sample, which of the following standard deviations would be most likely to result?
- A. 0.65°F
 - B. 0.73°F
 - C. 0.81°F
 - D. 0.89°F

29. Most computer languages have the ability to generate random numbers within a specified range. For example, the range could be specified to be between 0 and 100. The random numbers generated in this way exhibit a uniform frequency distribution. Which of the following graphs best represents this distribution?

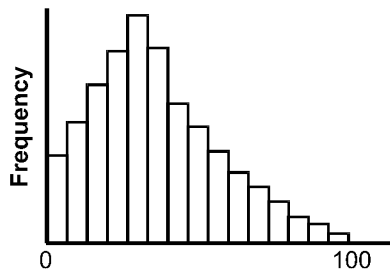
A.



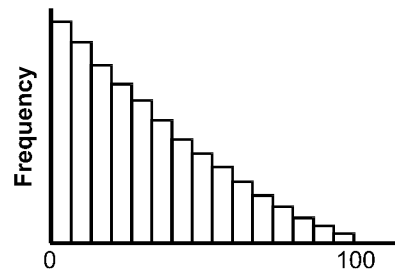
B.



C.



D.



30. There are 3 red marbles and 2 black marbles in a bag. What is the probability of drawing a black marble from the bag two times in a row if the marbles are replaced after each draw?

A. $\frac{1}{20}$

B. $\frac{1}{10}$

C. $\frac{4}{25}$

D. $\frac{2}{5}$

31. Two standard six-sided dice are rolled. What is the probability of rolling a two on at least one of the dice?

A. $\frac{1}{36}$

B. $\frac{1}{12}$

C. $\frac{11}{36}$

D. $\frac{1}{3}$

32. A jeweler creates a variety of earrings by choosing among n different stones, m different settings, and p different clasps. If each earring consists of one stone, one setting, and one clasp, how many different earrings can the jeweler make?

- A. $n \cdot m \cdot p$
- B. $3!(n + m + p)$
- C. $n + m + p$
- D. $n(m - 1)(p - 2)$

33. Use the figures below to answer the question that follows.

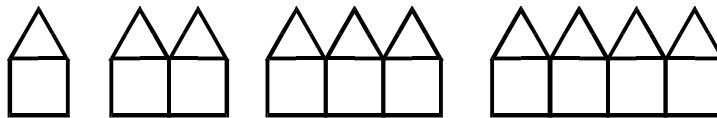


Figure 1

Figure 2

Figure 3

Figure 4

The length of a side of each square in the diagram above is 1 and the triangles are equilateral. If n represents the number of the figure, which of the following represents the perimeter of a figure as a function of its number?

- A. $2n + 2$
- B. $2n + 3$
- C. $3n + 2$
- D. $3n + 3$

34. Which of the following represents a possible domain and range of the function $f(x) = \sqrt{x}$?

A.

Domain	Range
1	1
4	2
9	3

B.

Domain	Range
0	0
2	4
3	9

C.

Domain	Range
1	1
8	2
27	3

D.

Domain	Range
2	1
4	2
6	3

35. Which of the following tables represents possible values for the dependent and independent variables of the function $f(x) = x^2 - 1$?

A.

Dependent	3	-1	0
Independent	-2	0	1

B.

Dependent	-5	-1	0
Independent	-2	0	1

C.

Dependent	-2	0	1
Independent	3	-1	0

D.

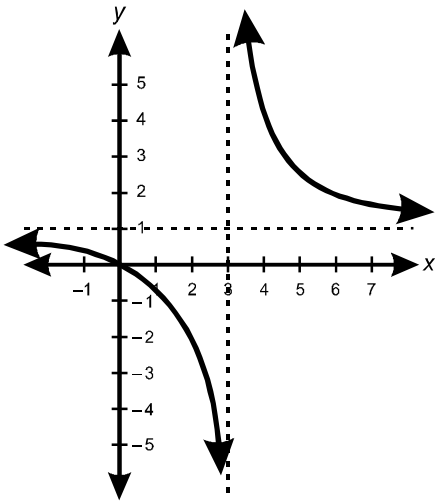
Dependent	-2	0	1
Independent	-5	-1	0

36. Use the table below to answer the question that follows.

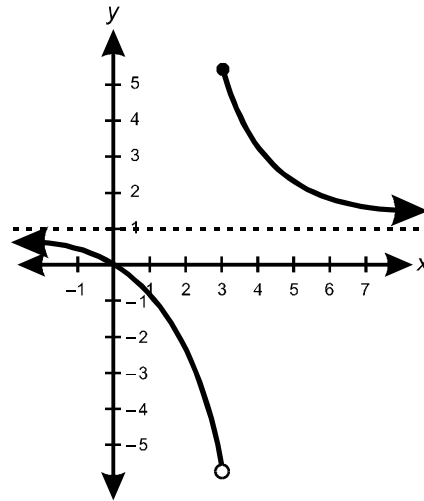
x	-10	-3	-1	1	3	5	10
y	0.97	0.80	0.50	-0.50	undefined	2.5	1.23

Which of the following graphs best represents the data in the table above?

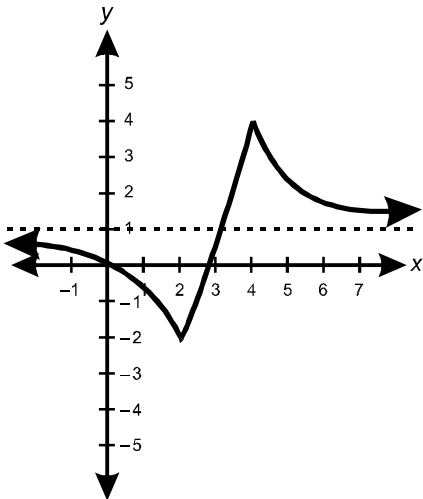
A.



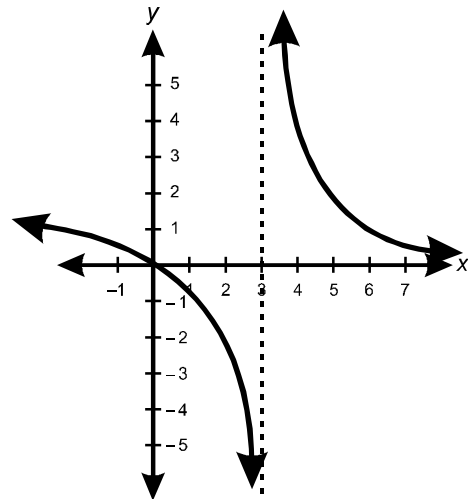
B.



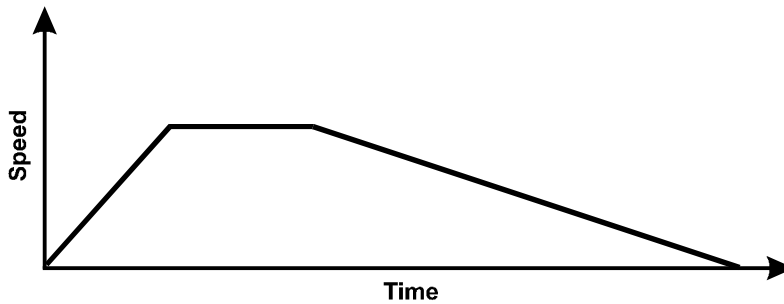
C.



D.



37. Use the qualitative graph below to answer the question that follows.



Which of the following scenarios best fits the qualitative graph above?

- A. A car accelerates to a maximum speed and then stops for a while. The car then turns around and slowly returns to its starting point.
- B. A bicycle travels to a certain point and then stops for a while. After stopping, the bicycle turns around and returns to its starting point.
- C. A bus accelerates until it reaches a certain speed. It maintains that speed for a while and then gradually slows down until it comes to a stop.
- D. A skateboard goes up a hill and then coasts on a level plateau. After coasting on the plateau, the skateboard rides down a ramp until it comes to a stop.

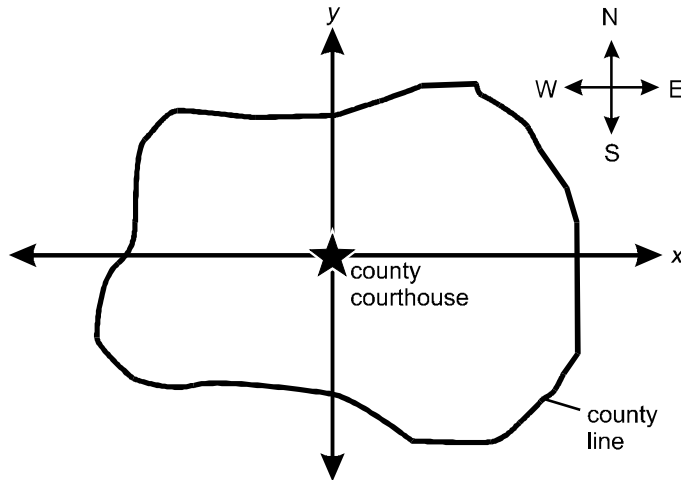
38. Use the expression below to answer the question that follows.

$$x + 1 - \frac{4}{x + 1}$$

Which of the following is equivalent to the expression above?

- A. $\frac{x - 3}{x + 1}$
- B. $\frac{x - 5}{x + 1}$
- C. $\frac{x^2 + x - 4}{x + 1}$
- D. $\frac{x^2 + 2x - 3}{x + 1}$

39. Use the map below to answer the question that follows.



A county map is put on an x - y grid as shown above. Every location is defined by an x -coordinate representing miles east or west of the county courthouse and a y -coordinate representing miles north or south of the county courthouse. County Road 14 is represented by the line $-2x + 3y = 10$. County Road 15 is a straight line perpendicular to County Road 14. County Road 15 passes through point $(4, -3)$. Which of the following equations represents County Road 15?

- A. $y = -\frac{3}{2}x + 3$
- B. $y = -\frac{2}{3}x - 3$
- C. $y = \frac{2}{3}x - 3$
- D. $y = \frac{3}{2}x + 3$

40. Use the table below to answer the question that follows.

Time	Location	Elevation
6:00 a.m.	Bright Angel Lodge	6860 feet
7:10 a.m.	Mile-and-a-Half Resthouse	5720 feet
8:00 a.m.	Three-Mile Resthouse	4920 feet

A group of people hikes down into the Grand Canyon starting at the Bright Angel Lodge. The table above gives three points on their hike, as well as the elevation of each location and the time the group is at the location. Assuming that they are descending at a constant rate, with no stops, which of the following equations best represents their elevation, y , in feet with respect to t , the number of hours since their start?

- A. $y = 16.2t + 4920$
- B. $y = -16.2t + 6860$
- C. $y = 970t + 4920$
- D. $y = -970t + 6860$

41. Use the information below to answer the question that follows.

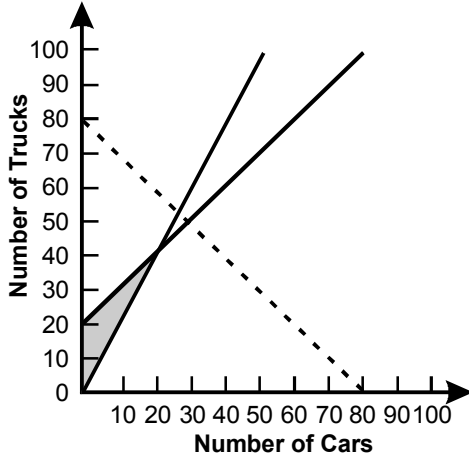
Day	Min. Temp.	Max. Temp.
Jan. 30	33°F	69°F
Jan. 31	38°F	75°F
Feb. 1	39°F	72°F
Feb. 2	37°F	75°F
Feb. 3	39°F	76°F
Feb. 4	36°F	78°F
Feb. 5	38°F	80°F
Feb. 6	29°F	79°F

The table above shows the minimum and maximum daily temperatures for Phoenix on eight consecutive days. On February 2 the minimum temperature occurred at 6:00 a.m. and the maximum temperature occurred at 3:00 p.m. If the temperature rose linearly during the day with respect to time, at which of the following times on February 2 was the temperature closest to 56°F?

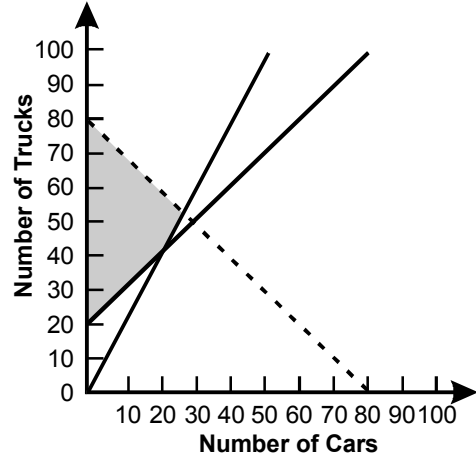
- A. 10:30 a.m.
- B. 11:30 a.m.
- C. 12:00 p.m.
- D. 1:00 p.m.

42. There are at least twice as many trucks as cars in a parking lot. The trucks outnumber the cars by 20 or more. The number of cars and trucks combined is less than 80. Which of the following graphs correctly represents the situation?

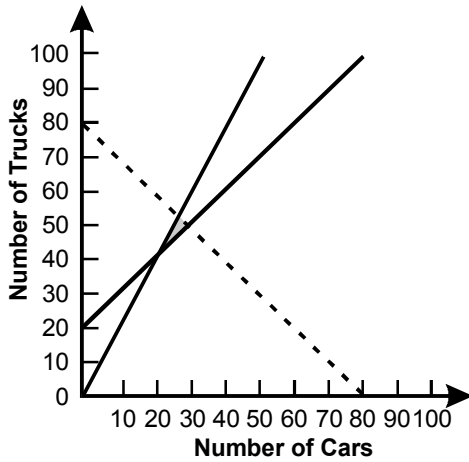
A.



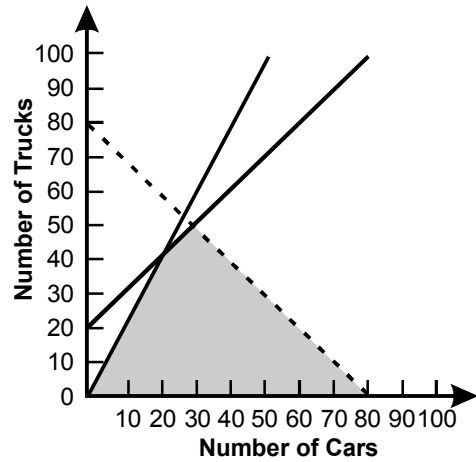
B.



C.



D.



43. Use the equations and inequalities below to answer the question that follows.

$$y_1 = m_1x + b_1$$

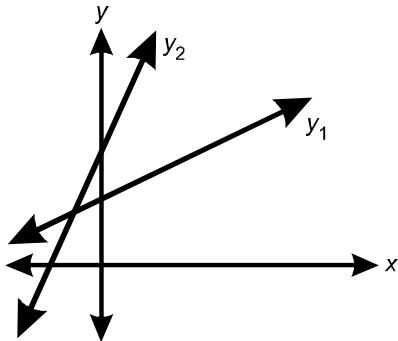
$$y_2 = m_2x + b_2$$

$$m_2 < m_1 < 0$$

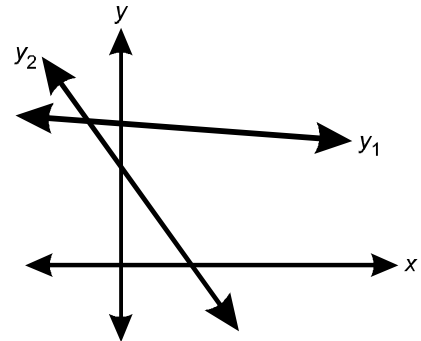
$$b_2 > b_1$$

Which of the following graphs represents y_1 and y_2 as functions of x ?

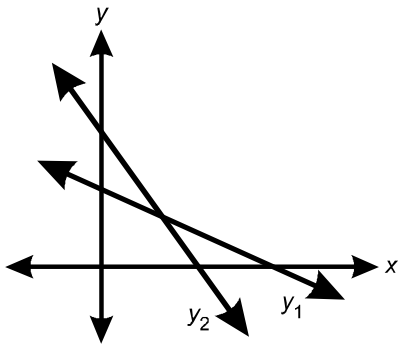
A.



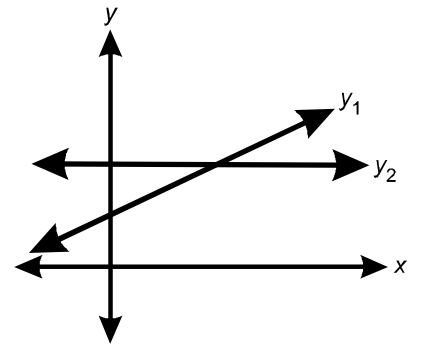
B.



C.



D.



44. Use the problem below to answer the question that follows.

After a rainfall the number of weeds growing in a garden increases by 50% each day. If the garden starts out with 20 weeds, approximately how many weeds will there be d days after a rainfall?

Which of the following expressions represents the solution to the problem above?

- A. $20(0.5)^d$
 - B. $20(1.5)^d$
 - C. $20 + 0.5d$
 - D. $20 + 1.5d$
45. Use the function below to answer the question that follows.

$$f(x) = \frac{1}{x - a}$$

As a increases, what is the effect on the graph of the function above?

- A. The vertical asymptote moves from left to right.
- B. The vertical asymptote moves from right to left.
- C. The horizontal asymptote moves from bottom to top.
- D. The horizontal asymptote moves from top to bottom.

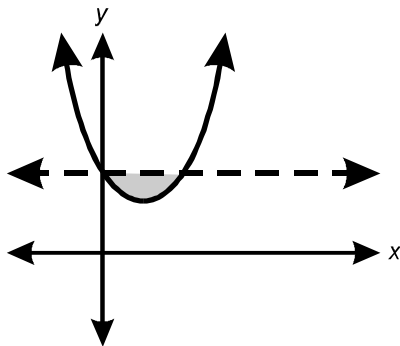
46. Use the system of inequalities below to answer the question that follows.

$$y \leq 2x^2 - 4x + 4$$

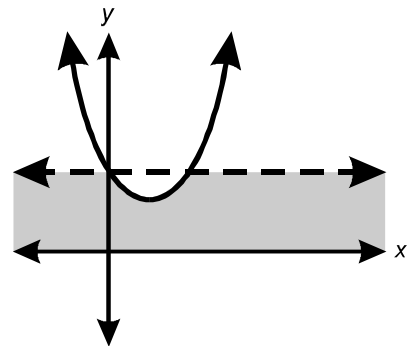
$$y < 4$$

Which of the following graphs represents the solution to the system of inequalities above?

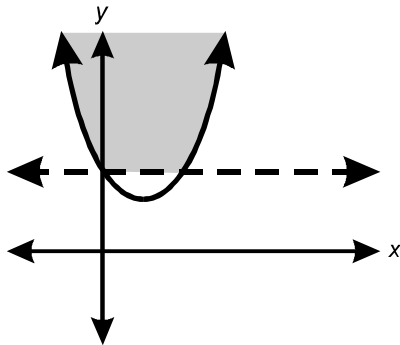
A.



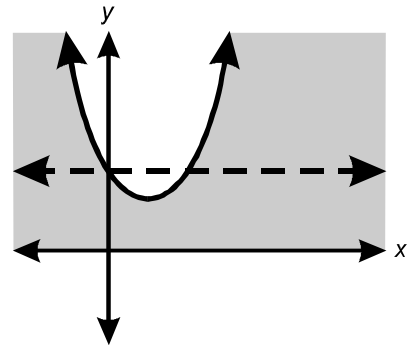
B.



C.



D.



47. Twice a college student's age is no more than four years different from any professor's age. If p represents any professor's age and s represents the student's age, which of the following inequalities represents the relationship between the age of the student and the age of a professor?

- A. $|2s - 4| \leq p$
- B. $|2s - p| \leq 4$
- C. $|p| - |2s| \leq 4$
- D. $|p - 4| \leq 2s$

48. **Use the equation below to answer the question that follows.**

$$2x^2 + 3x - 4 = 0$$

An engineer chooses to solve the equation above by using the quadratic formula rather than by factoring. Which of the following facts best explains this choice?

- A. The expression $3^2 - 4(2)(4)$ is negative.
- B. Only 3 is odd while 2 and -4 are even.
- C. The expression $3^2 - 4(2)(-4)$ is not a perfect square.
- D. Only one of the factor pairs of -4 adds to 3.

49. Typists for a professional typing service type 100 words per minute. The typing service charges \$2.00 per double-spaced page. If an average double-spaced page contains 250 words, which of the following calculations represents the amount of money the typing service charges in dollars per hour?
- A. $\frac{1}{100} \cdot 250 \cdot 2 \cdot 60$
- B. $100 \cdot \frac{1}{250} \cdot 2 \cdot 60$
- C. $100 \cdot 250 \cdot \frac{1}{2} \cdot 60$
- D. $100 \cdot 250 \cdot 2 \cdot \frac{1}{60}$
50. A right rectangular prism has length ℓ , height h , width w , and volume V_1 . If the height is increased by 5 units, which of the following expressions represents V_2 , the volume of the new right rectangular prism?
- A. $V_1 + 5h$
- B. $V_1 + 5\ell w$
- C. $V_1 + 5\ell wh$
- D. $V_1 + 25h$

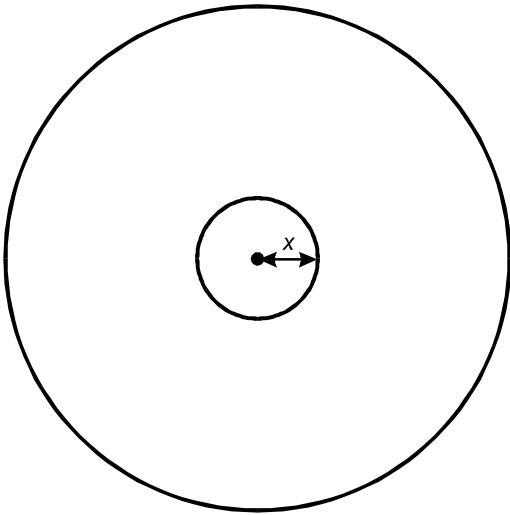
51. Use the expression below to answer the question that follows.

$$\frac{100 \text{ m}}{11 \text{ s}} \times \frac{39.37 \text{ in}}{1 \text{ m}} \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{1 \text{ mile}}{5280 \text{ ft}} \times \frac{3600 \text{ s}}{1 \text{ hour}}$$

Which of the following represents the final units in the conversion above?

- A. $\frac{\text{feet}}{\text{second}}$
- B. $\frac{\text{meters}}{\text{hour}}$
- C. $\frac{\text{inches}}{\text{second}}$
- D. $\frac{\text{miles}}{\text{hour}}$

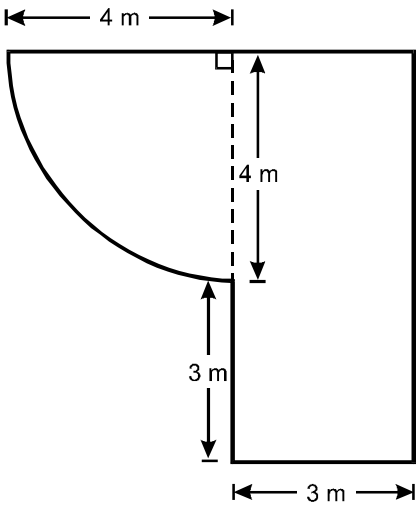
52. Use the diagram below to answer the question that follows.



The inner circle in the diagram above has radius x . The outer circle has a circumference of $8\pi x$. What is the radius of a circle that has a circumference equal to one third of the circumference of the outer circle?

- A. $\frac{4x}{3}$
- B. $\frac{2\sqrt{2}x}{3}$
- C. $2x$
- D. $2\sqrt{2}x$

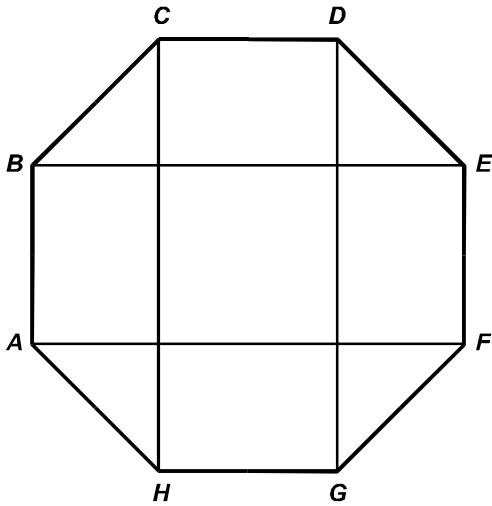
53. Use the diagram below to answer the question that follows.



A landscaper plans to surround the garden shown in the diagram above with a border of small decorative bricks. Each brick is 10 cm long. The bricks will be placed end-to-end in a single row around the perimeter of the garden. Approximately how many bricks will be needed to surround the garden?

- A. 260
- B. 300
- C. 450
- D. 530

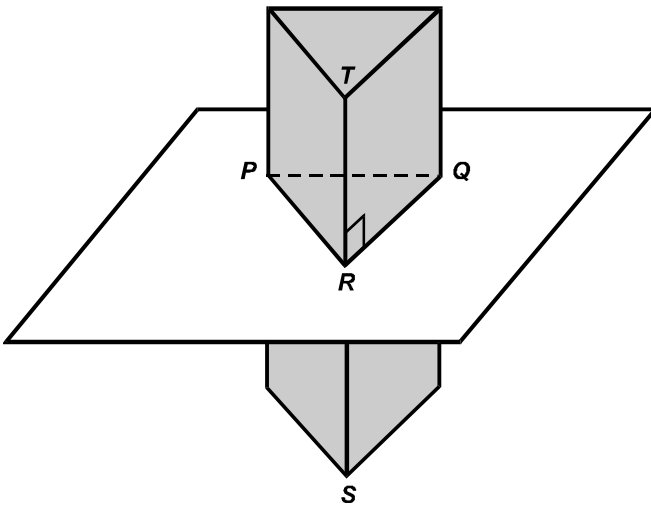
54. Use the diagram below to answer the question that follows.



$ABCDEFGH$ is a regular octagon. \overline{HA} has a length of 2 units.
What is the length of \overline{AF} ?

- A. $3\sqrt{2}$ units
- B. $4\sqrt{2}$ units
- C. $2 + \sqrt{2}$ units
- D. $2(1 + \sqrt{2})$ units

55. Use the diagram below to answer the question that follows.



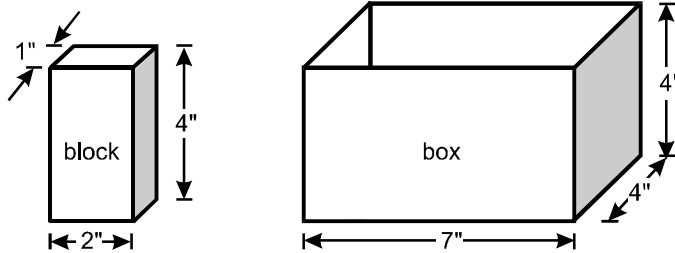
A plane intersects a triangular prism at an angle perpendicular to the vertical axis of the prism as shown above. If the plane rotates about \overline{PQ} so that R , its point of intersection with the opposite edge of the prism, moves along \overline{ST} , what happens to $\angle QPR$?

- A. $\angle QPR$ gets smaller, approaching 0°
- B. $\angle QPR$ gets smaller, approaching 45°
- C. $\angle QPR$ gets larger, approaching 90°
- D. $\angle QPR$ gets larger, approaching 180°

56. The lengths of the sides of a triangle from shortest to longest are x , y , and z respectively. The shortest side of a similar triangle has length a . Which of the following expressions represents the length of the longest side of the similar triangle?

- A. $\frac{x}{a}z$
- B. $\frac{a}{x}z$
- C. $\frac{y}{a}z$
- D. $\frac{a}{y}z$

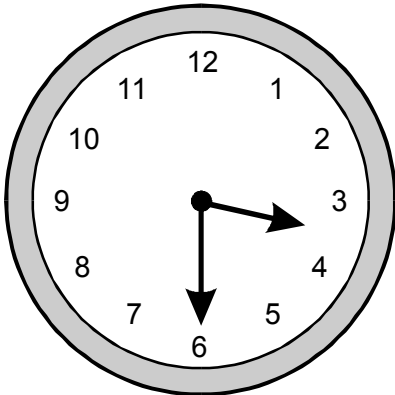
57. Use the diagram below to answer the question that follows.



What is the maximum number of blocks of dimensions $1'' \times 2'' \times 4''$ that can be packed inside the box of dimensions $4'' \times 4'' \times 7''$ shown above?

- A. 4
- B. 7
- C. 8
- D. 14

58. Use the clock below to answer the question that follows.

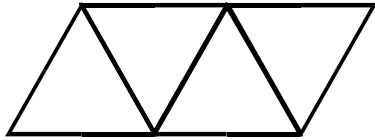


The clock above shows the time to be exactly 3:30. Which of the following expressions gives the degree measure of the angle between the hands of the clock?

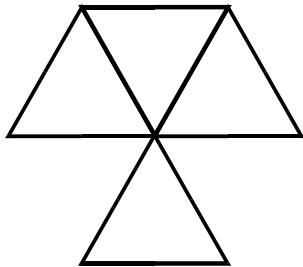
- A. $(360)\left(\frac{3+6}{12}\right)$
- B. $(360)\left(\frac{6-3}{12}\right)$
- C. $(360)\left(\frac{\left(3+\frac{6}{12}\right)+6}{12}\right)$
- D. $(360)\left(\frac{6-\left(3+\frac{6}{12}\right)}{12}\right)$

59. Which of the following diagrams represents a net for a tetrahedron?

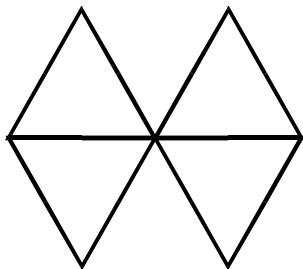
A.



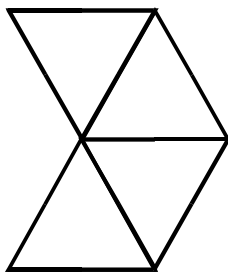
B.



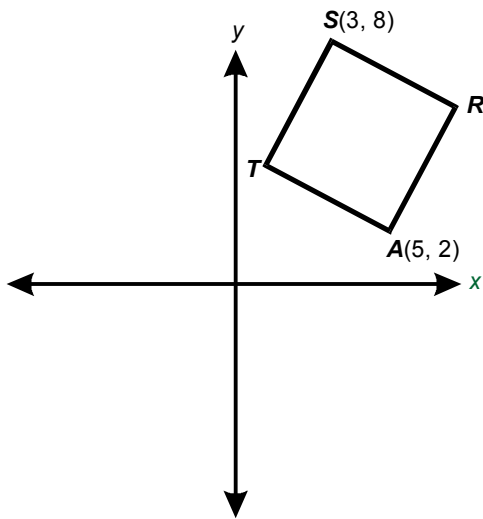
C.



D.



60. Use the diagram below to answer the question that follows.

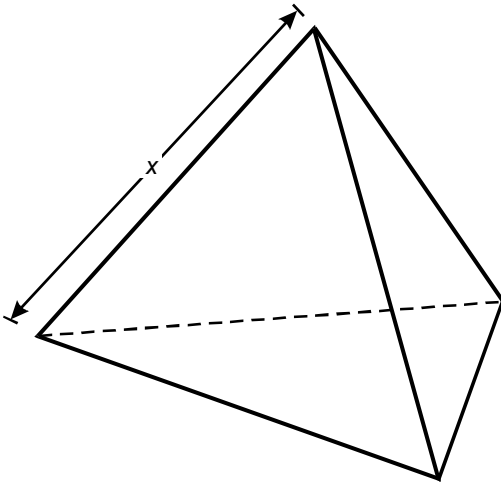


The points $S(3, 8)$ and $A(5, 2)$ are opposite vertices of square $STAR$. What is the slope of the line segment connecting points T and R ?

- A. -3
- B. $-\frac{1}{3}$
- C. $\frac{1}{3}$
- D. 3
61. Which of the following sets of points must represent a rhombus in the coordinate plane?
- A. $(2, 1), (2 + a, 1 + b), (2 + 3a, 1 + b), (2 + 4a, 1)$
- B. $(2, 1), (2 + a, 1 + b), (2 + a, 1 - b), (2 + 2a, 1)$
- C. $(2, 1), (2 + a, 1), (2, 1 - a), (2 + 2a, 1 - a)$
- D. $(2, 1), (2 + a, 1), (2, 1 - b), (2 + a, 1 - b)$

62. An isosceles triangle has base vertices at $(-2, 2)$ and $(-2, 8)$ and an apex at $(x, 5)$. For what value of x does the triangle have a perimeter equal to $6 + 6\sqrt{2}$?
- A. -3
B. -1
C. 1
D. 3
63. A triangle is represented by the points $(2, 1)$, $(5, 1)$, and $(3, 4)$ in the coordinate plane. Which of the following sets of points will represent the triangle after it is reflected across the x -axis?
- A. $(-2, 1)$, $(-5, 1)$, and $(-3, 4)$
B. $(-1, -2)$, $(-1, -5)$, and $(-4, -3)$
C. $(1, 2)$, $(1, 5)$, and $(4, 3)$
D. $(2, -1)$, $(5, -1)$, and $(3, -4)$
64. A group of friends traveling on a car trip first drives 200 km north, then 300 km east, and finally 400 km south. Which of the following expressions represents how many kilometers the friends end up from their starting point?
- A. $\sqrt{(300)^2 - (200)^2}$
B. $\sqrt{(300)^2 + (-200)^2}$
C. $\sqrt{(300 - 200)^2 + (200 - 400)^2}$
D. $\sqrt{(200)^2 + (300)^2} + \sqrt{(300)^2 + (400)^2}$

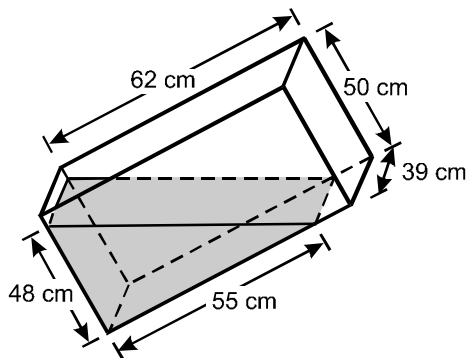
65. Use the diagram below to answer the question that follows.



The regular tetrahedron above is a pyramid with four equilateral triangles for faces. Which of the following formulas gives the surface area of the tetrahedron?

- A. $\frac{x^2}{8}\sqrt{3}$
- B. $\frac{x^2}{4}\sqrt{3}$
- C. $\frac{x^2}{2}\sqrt{3}$
- D. $x^2\sqrt{3}$

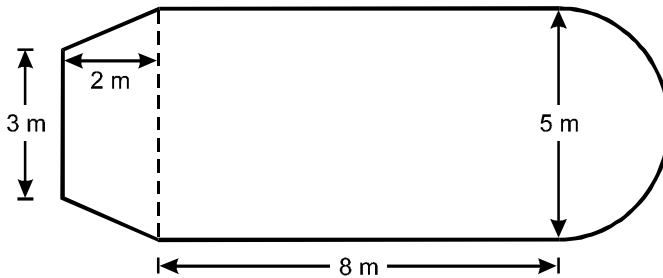
66. A storage rental facility has 1000 storage units. Of the storage units, 25% are small, 50% are medium, and 25% are large. On a particular day, 80% of the storage units are rented out. If 200 of the rented units are small and 150 of the rented units are large, how many medium storage units are still available to rent?
- A. 50
B. 150
C. 250
D. 350
67. Use the diagram below to answer the question that follows.



The diagram above shows a fish tank tilted up on one edge. Which of the following expressions represents the number of cubic centimeters of water in the fish tank?

- A. $48 \text{ cm} \cdot 39 \text{ cm} \cdot 55 \text{ cm}$
B. $48 \text{ cm} \cdot 39 \text{ cm} \cdot 62 \text{ cm}$
C. $\frac{1}{2}(48 \text{ cm} \cdot 39 \text{ cm} \cdot 55 \text{ cm})$
D. $\frac{1}{2}(48 \text{ cm} \cdot 39 \text{ cm} \cdot 62 \text{ cm})$

68. Use the diagram below to answer the question that follows.



A garden has the shape and dimensions shown in the diagram above. Which of the following expressions represents the area of the garden in square meters?

- A. $\frac{1}{2}(\pi \cdot 5^2) + (8 \cdot 5) + 2(3 + 5)$
- B. $\frac{1}{2}(\pi \cdot 2.5^2) + (8 \cdot 5) + 2(3 + 5)$
- C. $\frac{1}{2}(\pi \cdot 5^2) + (8 \cdot 5) + (3 + 5)$
- D. $\frac{1}{2}(\pi \cdot 2.5^2) + (8 \cdot 5) + (3 + 5)$

69. Use the word problem below to answer the question that follows.

A gardener wants to put a fence around a rectangular garden. The fencing material costs \$10 per meter. The gardener puts up the fence at a materials cost of \$200. What is the area of the garden?

What is the minimal amount of additional information needed to solve the word problem above?

- A. the length of one side of the garden
 - B. the lengths of two opposite sides of the garden
 - C. the lengths of two adjacent sides of the garden
 - D. the perimeter of the garden
70. Use the algorithm below to answer the question that follows.

Algorithm	
1	Choose a number.
2	Double it.
3	Move the decimal point one place to the left.

The algorithm above can be used for which of the following purposes?

- A. Divide a number by 5.
- B. Generate a random number.
- C. Convert a number to base 2.
- D. Find a number's smallest prime factor.

71. Use the theorem below to answer the question that follows.

Theorem: If a number is a positive integer, then it can be expressed as the sum of four or fewer squared numbers.

Which of the following statements follows from the theorem above?

- A. A negative integer cannot be expressed as the sum of four or fewer squared numbers.
 - B. Any number that cannot be expressed as the sum of four or fewer squared numbers is not a positive integer.
 - C. If a number can be expressed as the sum of four or fewer squared numbers, then the number is a positive integer.
 - D. There exist some numbers that are not positive integers, but that can be expressed as the sum of four or fewer squared numbers.
72. Use the statements below to answer the question that follows.

If A , then B . If B , then C .

Given the statements above, which of the following logical arguments is valid?

- A. A . Therefore, C .
- B. C . Therefore, A .
- C. B and C . Therefore, A .
- D. A or C . Therefore, B .

73. Use the logical argument below to answer the question that follows.

$$x + y = z \text{ and } a - b = y$$
$$\text{Therefore, } x + (a - b) = z.$$

The logical argument above is valid because:

- A. the converse statement and conclusion are also true.
 - B. the premise is clearly true, and therefore the conclusion must be true.
 - C. numbers can be inserted into the equations to make a true statement.
 - D. it is impossible for the premise to be true and the conclusion to be false.
74. Use the statements below to answer the question that follows.

Statement 1: All beaches are adjacent to water.
Statement 2: No deserts are beaches.

Which of the following follows logically from the two statements above?

- A. No beaches are deserts.
- B. All water is adjacent to a beach.
- C. No deserts are adjacent to water.
- D. Some beaches are adjacent to deserts.

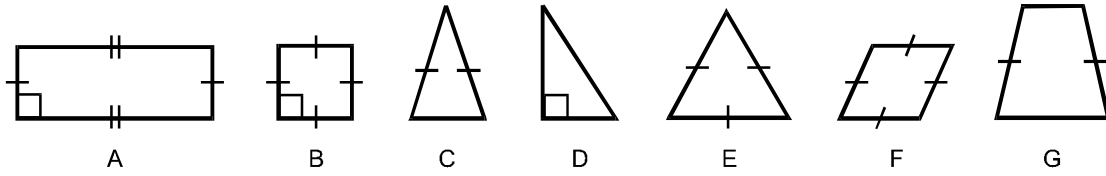
75. Use the statement below to answer the question that follows.

If n is even, then $\frac{2^n + n^2}{n}$ is a whole number.

Which of the following arguments demonstrates the use of a counterexample to evaluate the statement above?

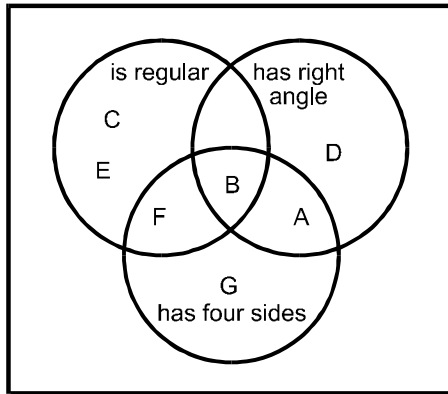
- A. $\frac{2^4 + 4^2}{4} = 8$ Therefore, the statement is true.
- B. $\frac{2^1 + 1^2}{1} = 3$ Therefore, the statement is false.
- C. $\frac{2^3 + 3^2}{3} = 5.67$ Therefore, the statement is false.
- D. $\frac{2^6 + 6^2}{6} = 16.67$ Therefore, the statement is false.
76. Students in an algebra class are investigating how changing the value of b in the equation $y = 2x + b$ affects the graph of the equation. This topic is most closely connected to which of the following topics in geometry?
- A. vertical lines
- B. concurrent lines
- C. perpendicular lines
- D. parallel lines

77. Use the shapes below to answer the question that follows.

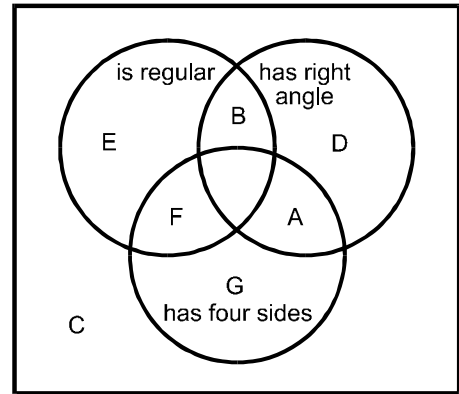


Which of the following Venn diagrams represents the classification of the shapes above?

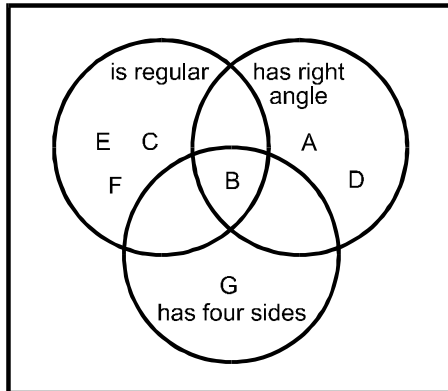
A.



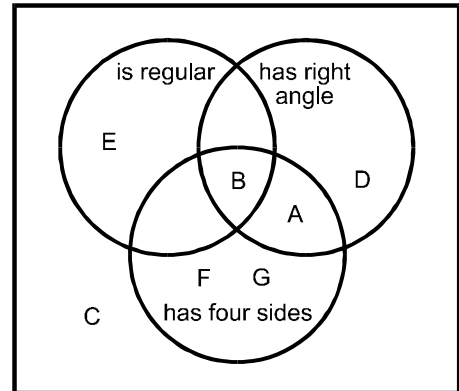
B.



C.



D.



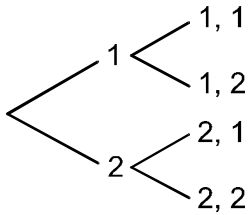
78. Use the grid below to answer the question that follows.

		Number on First Die					
		1	2	3	4	5	6
Number on Second Die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

The grid above can be used to calculate the probability of various outcomes when rolling two dice. For example, the probability that the sum of the two dice is 12 on a particular roll is $\frac{1}{36}$. This representation illustrates a connection between probability and which of the following other mathematical concepts?

- A. area
- B. logarithms
- C. map theory
- D. prime factorization

79. Use the diagram below to answer the question that follows.



- Which of the following is best illustrated by the diagram above?
- A. the properties of associative multiplication
 - B. the first two steps in creating a fractal tree
 - C. the concept of prime factorization in number theory
 - D. the possible outcomes after sequential random events
80. A piece of canvas material measuring $1\frac{3}{8}$ yards by 44 inches is used to make a tote bag. The material is trimmed to 48 inches by 44 inches. What is the approximate area of the material being cut away?
- A. 22 square inches
 - B. 26 square inches
 - C. 33 square inches
 - D. 66 square inches

Below are the directions for the Middle Grades Mathematics performance assignment.

DIRECTIONS FOR THE PERFORMANCE ASSIGNMENT

This section of the test consists of a performance assignment. **The assignment can be found on the next page.** You are asked to prepare a written response of approximately 2–3 pages on the assigned topic. You should use your time to plan, write, review, and edit your response for the assignment.

Read the assignment carefully before you begin to work. Think about how you will organize your response. You may use any blank space in this test booklet to make notes, write an outline, or otherwise prepare your response. **However, your score will be based solely on the version of your response written in Written Response Booklet B.**

As a whole, your response must demonstrate an understanding of the knowledge and skills of the field. In your response to the assignment, you are expected to demonstrate the depth of your understanding of the content area through your ability to apply your knowledge and skills rather than merely to recite factual information.

Your response will be evaluated based on the following criteria.

- **PURPOSE:** the extent to which the response achieves the purpose of the assignment
- **SUBJECT MATTER KNOWLEDGE:** accuracy and appropriateness in the application of subject matter knowledge
- **SUPPORT:** quality and relevance of supporting details
- **RATIONALE:** soundness of argument and degree of understanding of the subject matter

The performance assignment is intended to assess subject knowledge content and skills, not writing ability. However, your response must be communicated clearly enough to permit scorers to make a valid evaluation of your response according to the criteria listed above. Your response should be written for an audience of educators in this field. The final version of your response should conform to the conventions of edited American English. This should be your original work, written in your own words, and not copied or paraphrased from some other work.

Be sure to write about the assigned topic. Please write legibly. You may not use any reference materials during the test. Remember to review your work and make any changes you think will improve your response.

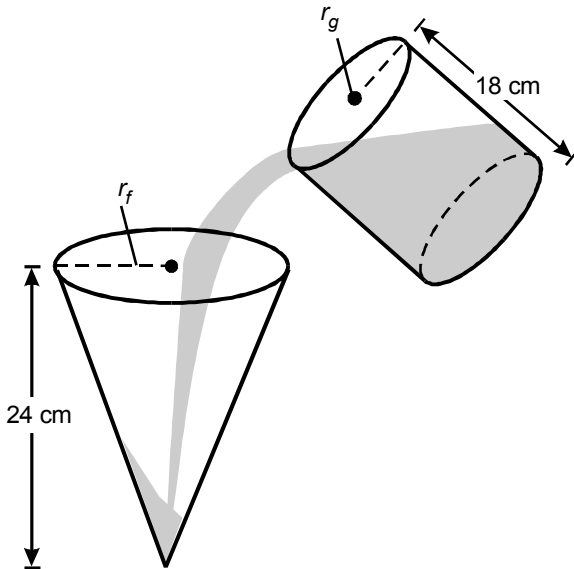
Below is the scoring scale for the Middle Grades Mathematics performance assignment.

SUBJECT TESTS—PERFORMANCE ASSIGNMENT SCORING SCALE

Score Point	Score Point Description
4	<p>The "4" response reflects a thorough knowledge and understanding of the subject matter.</p> <ul style="list-style-type: none"> • The purpose of the assignment is fully achieved. • There is a substantial, accurate, and appropriate application of subject matter knowledge. • The supporting evidence is sound; there are high-quality, relevant examples. • The response reflects an ably reasoned, comprehensive understanding of the topic.
3	<p>The "3" response reflects an adequate knowledge and understanding of the subject matter.</p> <ul style="list-style-type: none"> • The purpose of the assignment is largely achieved. • There is a generally accurate and appropriate application of subject matter knowledge. • The supporting evidence is adequate; there are some acceptable, relevant examples. • The response reflects an adequately reasoned understanding of the topic.
2	<p>The "2" response reflects a limited knowledge and understanding of the subject matter.</p> <ul style="list-style-type: none"> • The purpose of the assignment is partially achieved. • There is a limited, possibly inaccurate or inappropriate, application of subject matter knowledge. • The supporting evidence is limited; there are few relevant examples. • The response reflects a limited, poorly reasoned understanding of the topic.
1	<p>The "1" response reflects a weak knowledge and understanding of the subject matter.</p> <ul style="list-style-type: none"> • The purpose of the assignment is not achieved. • There is little or no appropriate or accurate application of subject matter knowledge. • The supporting evidence, if present, is weak; there are few or no relevant examples. • The response reflects little or no reasoning about or understanding of the topic.
U	<p>The response is unrelated to the assigned topic, illegible, primarily in a language other than English, not of sufficient length to score, or merely a repetition of the assignment.</p>
B	<p>There is no response to the assignment.</p>

Practice Performance Assignment

81. Use the information below to complete the exercise that follows.



A cylindrical glass full of water is poured into a conical filter as shown above. The water from the full glass is the exact volume needed to completely fill the filter. The glass has a height of 18 cm. The filter has a height of 24 cm.

Using your knowledge of geometry and algebra, write a response in which you:

- find the volume of the cylindrical glass as a function of r_g , the radius of the glass; and
- find the volume of the conical filter as a function of r_f , the radius of the filter; and
- find the ratio: $\frac{r_g}{r_f}$.

Be sure to show your work and explain the steps you used to find your answers.

Sample Performance Assignment Response: Score Point 4

To find the volume of the cylindrical glass, use the formula

$$V = \pi r^2 h \quad \left[\begin{array}{l} V = \text{volume} \\ r = \text{radius} \\ h = \text{height} \end{array} \right]$$

In this case the height is 18cm and the radius is represented as " r_g ".

So,

$$V = \pi r_g^2 18\text{cm} \text{ or}$$

$$V = (18\text{cm}) (\pi) r_g^2$$

To find the volume of the cone, the formula is $V = \frac{1}{3}\pi r^2 h$. Since the height is 24cm and the radius is represented as r_f

$$V = \frac{1}{3}\pi r_f^2 24\text{cm}$$

$$\text{or, } V = \frac{1}{3}(24\text{cm})(\pi) r_f^2 = (8\text{cm})(\pi) r_f^2$$

To find the ratio, $\frac{r_g}{r_f}$, first solve each equation for the radius.

Glass: $V = 18\text{cm}\pi r_g^2$

$$r_g^2 = \frac{V}{18\text{cm}\pi}$$

$$r_g = \pm \sqrt{\frac{V}{18\text{cm}\pi}}$$

(Since this is a "real" situation, the negative value can be ignored.)

Cone: $V = 8\text{cm}\pi r_f^2$

$$r_f^2 = \frac{V}{8\text{cm}\pi}$$

$$r_f = \pm \sqrt{\frac{V}{8\text{cm}\pi}}$$

(Again, the negative value is not useful.)

(continued on next page)

Sample Performance Assignment Response: Score Point 4 (continued)

So,

$$\frac{r_g}{r_f} = \frac{\sqrt{\frac{V}{18cm\pi}}}{\sqrt{\frac{V}{8cm\pi}}}$$

To simplify this can be written

$$\begin{aligned}\frac{r_g}{r_f} &= \sqrt{\frac{V}{18cm\pi}} \times \sqrt{\frac{8cm\pi}{V}} \\ &= \sqrt{\frac{V}{18cm\pi} \times \frac{8cm\pi}{V}}\end{aligned}$$

reducing under the radical gives

$$\frac{r_g}{r_f} = \sqrt{\frac{8}{18}} = \sqrt{\frac{4}{9}} = \pm \frac{2}{3}$$

(Again, the negative is not useful.)

So,

$$\frac{r_g}{r_f} = \frac{2}{3}$$

ANSWER KEY

Field 37: Middle Grades Mathematics

Question Number	Correct Response	Objective
1.	C	Understand the structure of numeration systems and ways of representing numbers.
2.	B	Understand the structure of numeration systems and ways of representing numbers.
3.	B	Understand the structure of numeration systems and ways of representing numbers.
4.	C	Understand the structure of numeration systems and ways of representing numbers.
5.	D	Understand the structure of numeration systems and ways of representing numbers.
6.	C	Understand the structure of numeration systems and ways of representing numbers.
7.	D	Understand numerical operations and properties of the real number system.
8.	B	Understand numerical operations and properties of the real number system.
9.	A	Understand numerical operations and properties of the real number system.
10.	C	Understand numerical operations and properties of the real number system.
11.	A	Understand numerical operations and properties of the real number system.
12.	B	Understand the principles of number theory.
13.	C	Understand the principles of number theory.
14.	D	Understand the principles of number theory.
15.	A	Understand the principles of number theory.
16.	B	Understand the principles of number theory.
17.	B	Understand methods of collecting, organizing, and displaying data.
18.	B	Understand methods of collecting, organizing, and displaying data.
19.	D	Understand methods of collecting, organizing, and displaying data.
20.	C	Understand methods of collecting, organizing, and displaying data.
21.	B	Understand methods of collecting, organizing, and displaying data.
22.	D	Understand methods of collecting, organizing, and displaying data.
23.	A	Understand methods of describing and analyzing data.
24.	B	Understand methods of describing and analyzing data.
25.	B	Understand methods of describing and analyzing data.
26.	D	Understand methods of describing and analyzing data.
27.	B	Understand methods of describing and analyzing data.
28.	A	Understand methods of describing and analyzing data.
29.	B	Understand the fundamental principles of probability, probability distributions, and discrete mathematics.
30.	C	Understand the fundamental principles of probability, probability distributions, and discrete mathematics.

(continued on next page)

Question Number	Correct Response	Objective
31.	C	Understand the fundamental principles of probability, probability distributions, and discrete mathematics.
32.	A	Understand the fundamental principles of probability, probability distributions, and discrete mathematics.
33.	C	Understand patterns, relations, algebraic functions, and expressions.
34.	A	Understand patterns, relations, algebraic functions, and expressions.
35.	A	Understand patterns, relations, algebraic functions, and expressions.
36.	A	Understand patterns, relations, algebraic functions, and expressions.
37.	C	Understand patterns, relations, algebraic functions, and expressions.
38.	D	Understand patterns, relations, algebraic functions, and expressions.
39.	A	Understand and use linear functions to model and solve problems.
40.	D	Understand and use linear functions to model and solve problems.
41.	A	Understand and use linear functions to model and solve problems.
42.	B	Understand and use linear functions to model and solve problems.
43.	C	Understand and use linear functions to model and solve problems.
44.	B	Understand and use nonlinear functions and relations to model and solve problems.
45.	A	Understand and use nonlinear functions and relations to model and solve problems.
46.	B	Understand and use nonlinear functions and relations to model and solve problems.
47.	B	Understand and use nonlinear functions and relations to model and solve problems.
48.	C	Understand and use nonlinear functions and relations to model and solve problems.
49.	B	Understand principles and procedures related to measurement.
50.	B	Understand principles and procedures related to measurement.
51.	D	Understand principles and procedures related to measurement.
52.	A	Understand principles and procedures related to measurement.
53.	A	Understand principles and procedures related to measurement.
54.	D	Understand principles of Euclidean geometry and properties of two- and three-dimensional figures.
55.	C	Understand principles of Euclidean geometry and properties of two- and three-dimensional figures.
56.	B	Understand principles of Euclidean geometry and properties of two- and three-dimensional figures.
57.	D	Understand principles of Euclidean geometry and properties of two- and three-dimensional figures.
58.	D	Understand principles of Euclidean geometry and properties of two- and three-dimensional figures.

(continued on next page)

Question Number	Correct Response	Objective
59.	A	Understand principles of Euclidean geometry and properties of two- and three-dimensional figures.
60.	C	Understand principles of coordinate and transformational geometry.
61.	B	Understand principles of coordinate and transformational geometry.
62.	C	Understand principles of coordinate and transformational geometry.
63.	D	Understand principles of coordinate and transformational geometry.
64.	B	Understand principles of coordinate and transformational geometry.
65.	D	Understand principles of problem solving.
66.	A	Understand principles of problem solving.
67.	C	Understand principles of problem solving.
68.	D	Understand principles of problem solving.
69.	A	Understand principles of problem solving.
70.	A	Understand principles of problem solving.
71.	B	Understand principles of mathematical reasoning.
72.	A	Understand principles of mathematical reasoning.
73.	D	Understand principles of mathematical reasoning.
74.	A	Understand principles of mathematical reasoning.
75.	D	Understand principles of mathematical reasoning.
76.	D	Understand the techniques for communicating mathematical information and connections within and outside the domains of mathematics.
77.	D	Understand the techniques for communicating mathematical information and connections within and outside the domains of mathematics.
78.	A	Understand the techniques for communicating mathematical information and connections within and outside the domains of mathematics.
79.	D	Understand the techniques for communicating mathematical information and connections within and outside the domains of mathematics.
80.	D	Understand the techniques for communicating mathematical information and connections within and outside the domains of mathematics.

PREPARATION RESOURCES

Field 37: Middle Grades Mathematics

The resources listed below may help you prepare for the AEPA® test in this field. These preparation resources have been identified by content experts in the field to provide up-to-date information that relates to the field in general. You may wish to use current issues or editions to obtain information on specific topics for study and review.

Online Sources:

Arizona Collaborative for Excellence in the Preparation of Teachers. Arizona State University.
<http://accept.la.asu.edu>

Arizona Department of Education. Content Standards: Mathematics Standard Articulated by Grade Level.
<http://www.azed.gov/standards/math/articulated.asp>

Arizona Department of Education. Math Resource Page
<http://www.azed.gov/SBTL/sdi/math.asp>

Coconino County Math and Science Teachers Resources
<http://co.coconino.az.us/schools.aspx?id=534>

Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics
<http://www.nctm.org/focalpoints/>

The Math Forum. Drexel University
<http://www.mathforum.org>

National Council of Teachers of Mathematics
<http://www.nctm.org>

TODOS: Mathematics for All
<http://www.todos-math.org>

Journals:

Teaching Children Mathematics, National Council of Teachers of Mathematics.

Mathematics Teacher, National Council of Teachers of Mathematics.

Mathematics Teaching in the Middle School, National Council of Teachers of Mathematics.

Other Sources:

Billstein, R., Libeskind, S., & Lott, J. *A Problem Solving Approach to Mathematics for Elementary School Teachers*, 8th ed. Boston: Addison Wesley, 2004.

Blitzer, R. *Introductory Algebra*, 4th ed. Upper Saddle River, NJ: Prentice Hall, 2006.

Charles, R. I., et.al. *Mathematics: Grade 6*. Glenview, IL: Scott Foresman, 2004.

Lial, M., Brown, B., Steffenson, A., & Johnson, L. *Essentials of Geometry for College Students*, 2nd ed. Boston: Addison Wesley, 2004.

Musser, G. L., Burger, W. F., & Peterson, B. E., *Essentials of Mathematics for Elementary Teachers: A Contemporary Approach*, 6th ed. Hoboken, NJ: John Wiley, 2003.

Parker, T., & Baldrige, S. *Elementary Mathematics for Teachers (Vol. 1)*, Okemos, MI: Sefton-Ash Publishing, 2004.

Principles and Standards for School Mathematics, Reston, VA: National Council of Teachers of Mathematics, Inc., 2000.

University of Chicago Mathematics Project. *Advanced Algebra*. Upper Saddle River, NJ: Prentice Hall, 2002.

University of Chicago Mathematics Project. *Transition Mathematics*. Upper Saddle River, NJ: Prentice Hall, 2002.